

Functions and the Common Core:

$$f(\text{CCSS}) = ???$$

Implications for Planning and Teaching

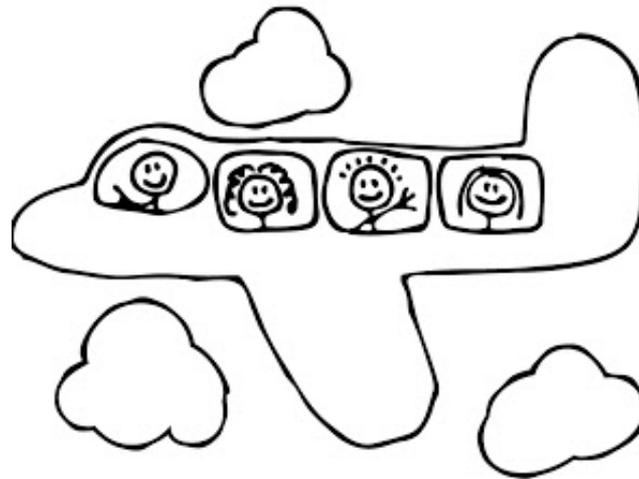
A workshop prepared for the
Rhode Island Department of Education

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Goals for Today:

- Develop an understanding of the progression of the CCSS standards within the function domains for Grade 8 & Algebra I
- Develop a deeper understanding of the function concept to inform our instruction
- Focus on the types of experiences students should have in grades 6 – 9 in order to fully “grasp” the concept of function
- Explore resources for rich classroom activities aimed at developing function sense that incorporate the Standards for Mathematical Practice

Overview of Function Domains



New Domain “Functions”(Gr 8):

- ccss **Critical** Area



Function Domain (Gr 8):

- 5 Standards (8.F)

Students –

- Develop an intuitive understanding of what a function is.
- Explore functional relationships in equations, tables, graphs, and verbal descriptions.
- Emphasis is on linear models.

High School - Algebra and Functions

Two separate conceptual categories that are interrelated -

- Algebra category contains most of the typical “symbol manipulation” standards
- Functions category is more conceptual

Shift from GLEs/GSEs

- GLE's for "Functions & Algebra":
 - F&A.1: Identifying and extending **patterns**
 - F&A.2: Demonstrating conceptual understanding of **linear relationships**
 - F&A.3: Demonstrating conceptual understanding of **algebraic expressions**
 - F&A.4: Demonstrating conceptual understanding of **equality**

4 Function Domains in High School

- Interpreting functions (F-IF.1-9)
- Building Functions (F-BF.1-5)
- Linear, Quadratic, and Exponential models (F-LE.1-5)
- Trigonometric Functions (F-TF.1-9)

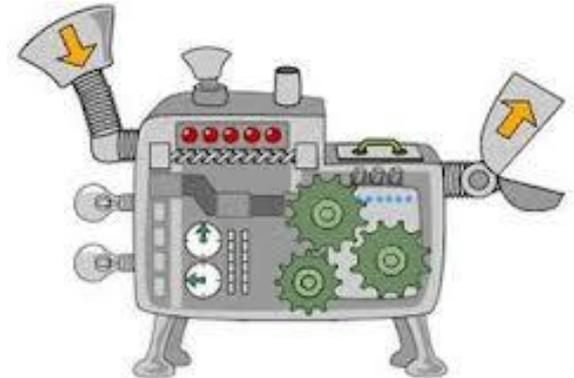
What is “Function Sense”?

- Complex notion
- You might have noticed that providing students with the definition doesn't do it.
- In groups of 4 or 5, on poster paper...



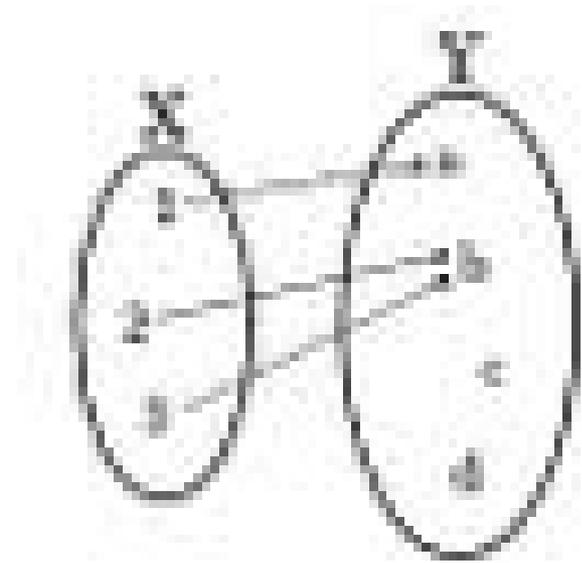
SO MANY "ASPECTS" OF FUNCTION

- **Dependence relationship**
- **Rule/machine**
- **Set of ordered pairs**
- **Varying quantities**
- **Assignment (mapping)**



SO MANY “REPRESENTATIONS” OF FUNCTION

- Equation ($y=$ or $f(x)=$)
- Graph
- Rule
- Table
- Set of Points



Function Sense Indicator:

- Flexibility in transforming between alternate representations.



A well-developed sense for functions is demonstrated by the ability to tie together these various representations.

- **See 8.F.2, F-IF.8-9**

Mathematical Practice I

“Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends”

(CCSS (2010), p. 6)

Function Sense Indicator:

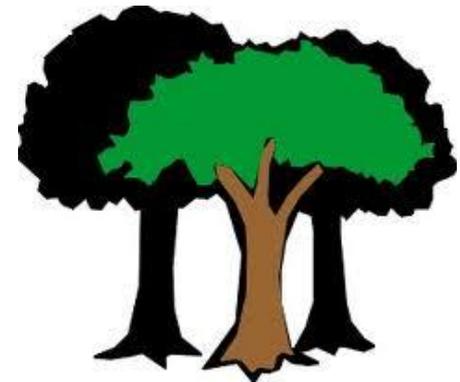
- Appreciation for and ability to apply function concept and procedures to real world settings (internalizing ideas)

More than input and output or a “recipe” - know what’s changing, how it’s changing, can handle point-wise questions as well as global (across-time) questions....focus on quantities and their relationships instead of number and arithmetic operations.

See the forest for the trees!

“Consider the traditional approach...Most instruction does not take the next step, that is, to advance from the generating and plotting of specific points to a global inspection of the resultant graph. In other words, students are so engaged with the mechanics of generating and plotting individual points, that, once completed, they never “step back” to view the pattern produced by the points. They never learn to see the forest for the trees!”

(Stein, Leinhardt, 1989)



A Technological World:

- Away from algebra as rules for transforming, simplifying and solving symbolic expressions to algebra as a way to express and analyze **RELATIONSHIPS**.

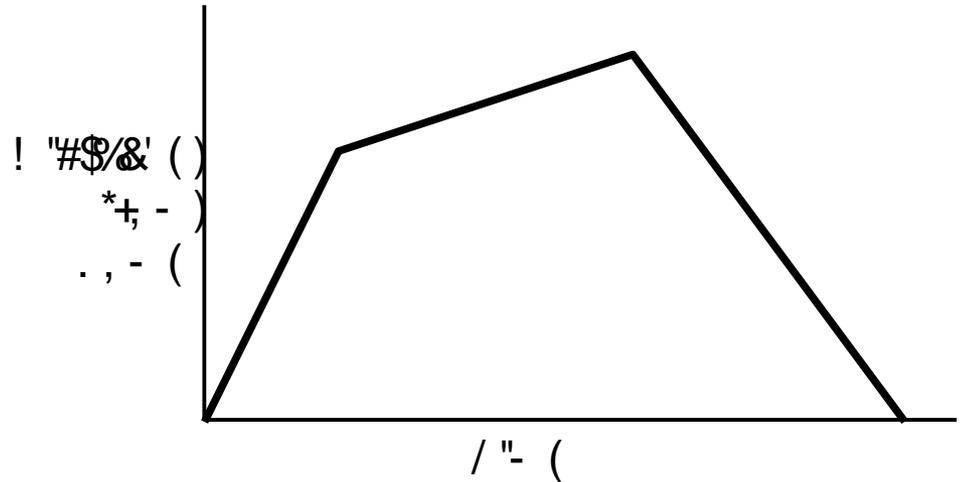


Matching a Graph to a Story

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



Function Sense

- If we are successful in getting our students to develop a certain level of fluency and ease and comfort with functions, and their graphs and interpretations we can say that they have developed a reasonable level of “*function sense*”.

The task for instruction:

- Task for instruction and for learning is to expand and pull together these seemingly disparate threads into a unified, mature notion of function.



Implications for instruction:

Are we afraid that this might just confuse students?

- Coming out of this confusion is what builds mathematical power and forms insights.
- Amount of time we think it takes to learn the concept may be way off.

Implication for instruction:

- Abstract and formal approaches should not be rushed.
- Current textbook emphasis?

Critical Foundations for Developing “Function Sense”

- Number Sense
- Operation Sense
- Symbol/Variable Sense
- Expression Sense
- Graph Sense

Number Sense

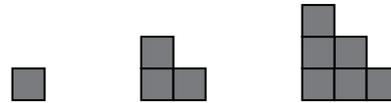
- Flexibility with numbers
- Recognizing number characteristics and relationships between numbers



Patterns

- Number patterns 1, 4, 9, 16, 25, __, __, ...

- Growing patterns



- ✓ **3.OA.9:** Identifying arithmetic patterns
- ✓ **4.OA.5:** Generating number or shape pattern that follows a rule
- ✓ **5.OA.3:** Identify relationships between corresponding terms in two patterns



Quote from GLEs:

“The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions.”

(Rhode Island K-8 Mathematics Grade Level Expectations, p. 19)

The problem with patterns:

Students are asked to continue the pattern 2, 4, 6, 8, Here are some legitimate responses:

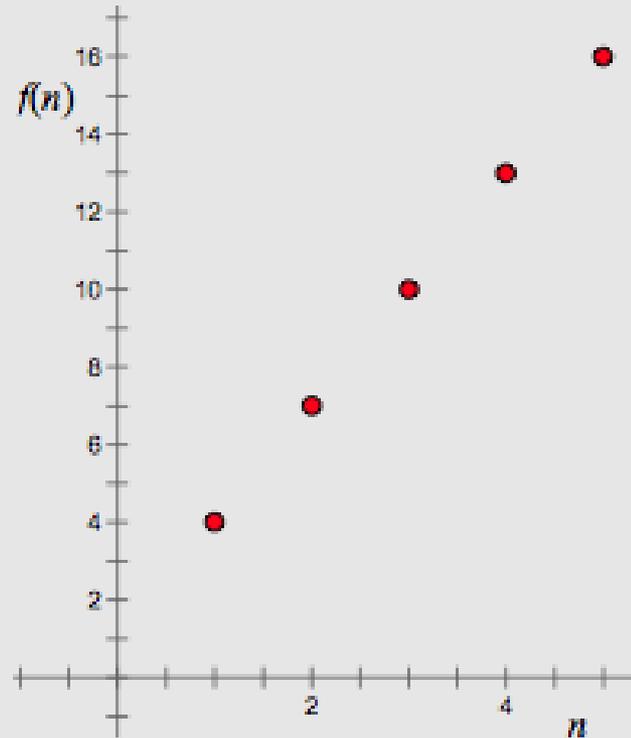
- Cody: I am thinking of a “plus 2 pattern,” so it continues 10, 12, 14, 16,
- Ali: I am thinking of a repeating pattern, so it continues 2, 4, 6, 8, 2, 4, 6, 8,
- Suri: I am thinking of the units digit in the multiples of 2, so it continues 0, 2, 4, 6, 8, 0, 2,
- Erica: If $g(n)$ is any polynomial, then $f(n) = 2n + (n - 1)(n - 2)(n - 3)(n - 4)g(n)$ describes a continuation of this sequence.
- Zach: I am thinking of that high school cheer, “Who do we appreciate?”

The problem with patterns:

- Without any structure, continuing the pattern is simply speculation – a guessing game. Because there are infinitely many ways to continue a sequence, patterning problems should provide enough structure so that the sequence is well defined.

Sequences as functions

n	$f(n)$
1	4
2	7
3	10
4	13
5	16



With shoulder partner:

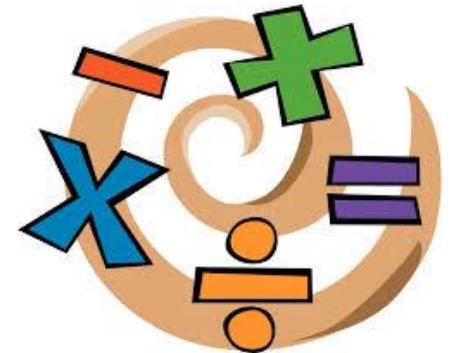
- Describe the recursive relationship. How would we write that using function notation?
- What would be an explicit (closed) formula?

Sequences (F-IF.3, F-BF.2)

- Describe “recursively” - requires starting value and a rule for subsequent terms
- Describe with a “closed” or “explicit” formula (domain is included)
- Graph consists of discrete dots.

Operation Sense

- Understanding the meanings for each operation
- Understanding the relationships between operations
- Knowing when an operation is useful



Variable Sense

- Being able to use a variable to generalize a pattern
- Being able to use a variable as an unknown to solve a problem
- Understanding joint variation (that a change in one variable may cause a change in another variable)



Leap to abstraction!

- Several meanings and aspects of variable:

STATIC vs. DYNAMIC

ABSTRACT vs. CONTEXTUALIZED

DISCRETE vs. CONTINUOUS

- These characteristics are not generally addressed in instruction and are left for student to induce.
- We could do a better job at articulating these characteristics of variable.

Developing Variable Sense

K.OA: Using objects or drawings to represent problem

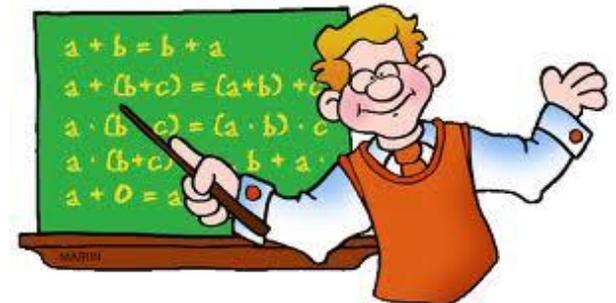
1.OA, 2.OA, 3.OA: Using symbol for the unknown number

4.OA: Letter standing for unknown quantity

5.OA: Use parentheses, brackets, or braces in numerical expressions

Expression Sense

- Being able to translate English language to mathematical language and vice versa
- Recognizing equivalent forms for simple algebraic expressions, i.e. $2x = x + x$
- Being able to use the properties of operations to obtain an equivalent expression
- Understanding the equals sign



Variable & Expression Sense

You know that there are 3 ft in a yard.

Using F for feet and Y for yards, write an equation that expresses this relationship.

Many students will write $3F = Y$. If I had said use F for number of feet and Y for number of yards, you might have had more success. (Saying “ F for feet and Y for yards” made it seem that these variables were labels rather than numbers.)



Variable & Expression Sense

1. $4 + 0.75 = 4.75$, but $x + y \neq xy$;
2. $a \times b = ab$ but $3 \times 5 \neq 35$;
3. $ab = ba$ but $35 \neq 53$

Variable & Expression Sense



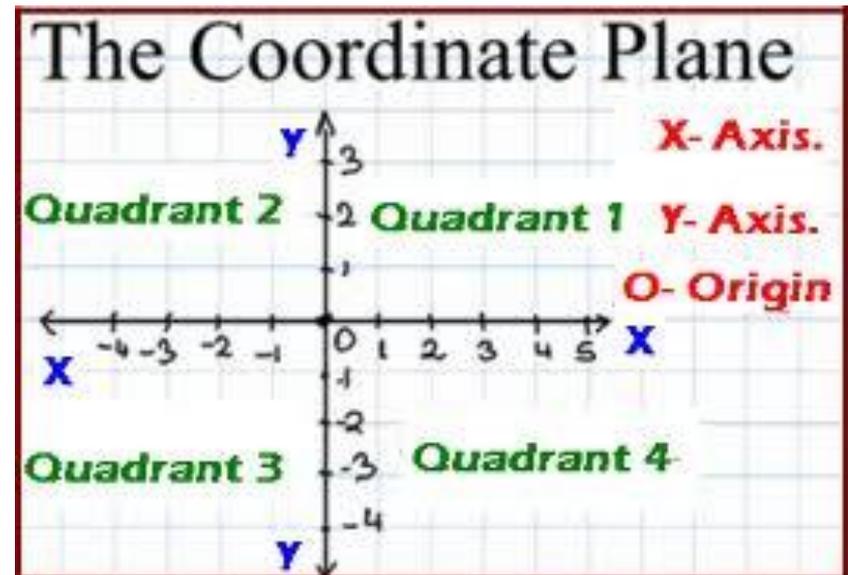
"Just a darn minute! — Yesterday
you said that X equals two!"

Variable & Expression Sense

*Dear Algebra,
Stop asking us to find your X
She's not coming back*

Graph Sense

- Being familiar with the coordinate plane
- Being able to use a graph to describe relationships between variables and make predictions

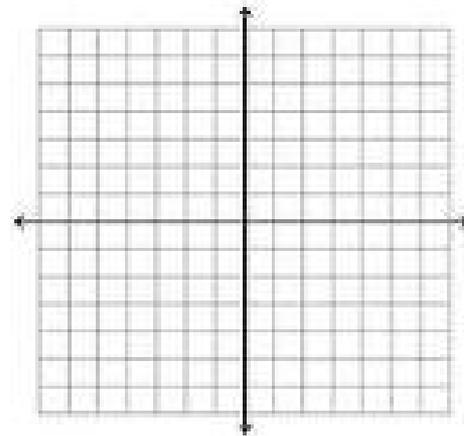


Graph Sense

- 5.G: Graph points on the coordinate plane to solve real-world and mathematical problems
- 6.G: Draw polygons in the coordinate plane
- 7.RP: Graphing proportional relationships and identifying constant of proportionality (unit rate) from the graph, equation, table, diagram, or verbal description.

Activities to Develop Graph Sense

- Fitting Graph to Story or vice versa (see 8.F.5)
- Real world activities: Scatterplots / best fit lines (8.SP.2)



Link to Statistics & Probability

8.SP.2:

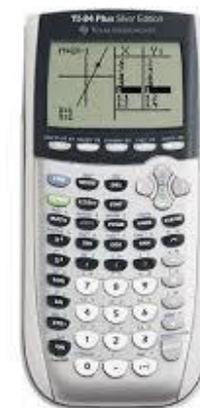
- Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Difficulties (intertwined with previous learning):

- Students see what they are prepared to see (missed or blurred concepts)
- Over-literal interpretations of graphs
- See static picture vs. dynamic process
- Scaling can cause misconceptions as it affects shape

Graphing calculator:

- Widely claimed that a graphing calculator can help students develop a deeper understanding of and appreciation for functions (research is starting to support this) - esp. improved ability to conceptually connect alternate representations of functional ideas.
- Changes roles in classroom - student is explorer, evaluator, explainer... more conjecturing and analyzing...





Standards for Mathematical Practice

**Mathematically
Proficient
students...**

Overarching habits of mind
of a productive mathematical thinker.

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

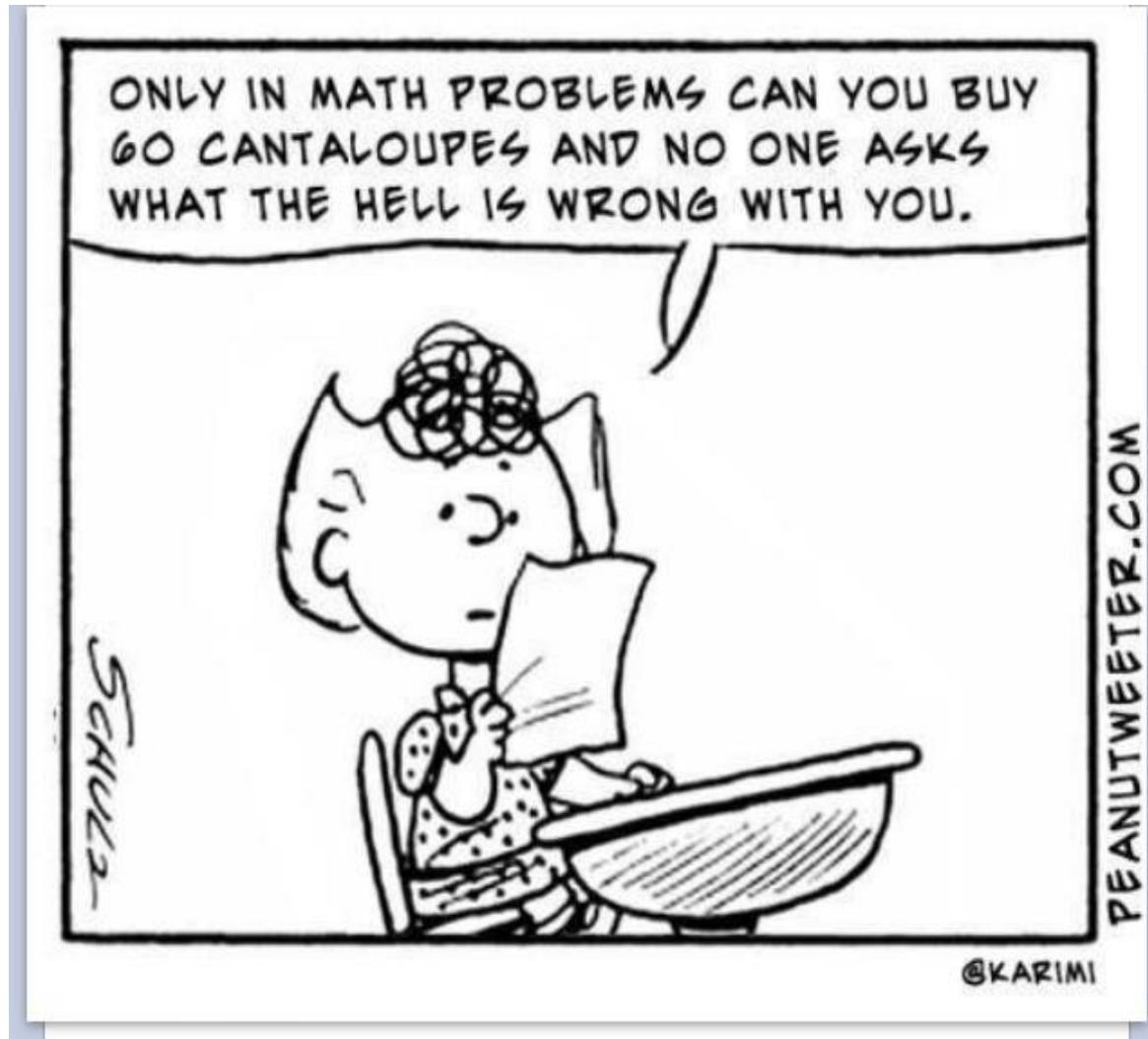
8. Look for and express regularity in repeated reasoning.

Reasoning and explaining

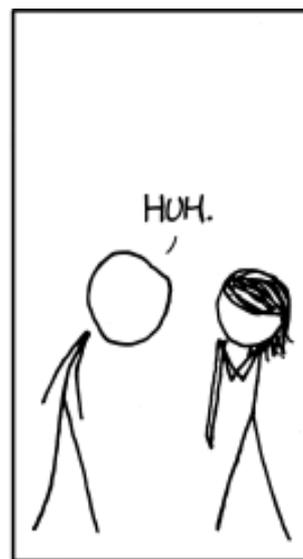
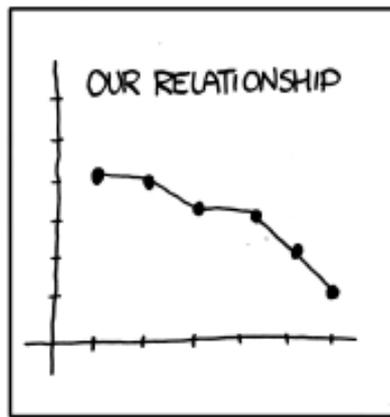
Modeling and using tools

Seeing structure and generalizing

MPI: Make sense of problems and persevere in solving them.



MP 6: Attend to precision



Definitions of “Function”

- Some common uses of the word are quite different from the mathematical meaning
 - “Body function”
 - “Meal function”
- Other uses of word are close to mathematical meaning
 - Number of cavities is a function of how often you brush your teeth



Definitions of “Function”

See 8.F.1

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹

(¹Function Notation not required in Grade 8)

Definitions of “Function”

See I-IF.1

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.



If one of your students asked you to explain the difference between a function and an equation, how would you respond?

Developing Essential Understanding of Expressions, Equations and Functions (Grades 6-8), NCTM

Notation:

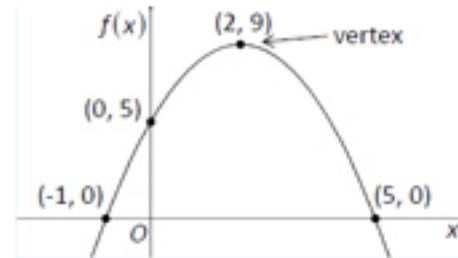
See F-IF.2

- To promote fluency with function notation, students should interpret function notation within contexts.

PARCC items

SAMPLE ITEM

A portion of the graph of a quadratic function $f(x)$ is shown in the xy - plane. Selected values of a linear function $g(x)$ are shown in the table.



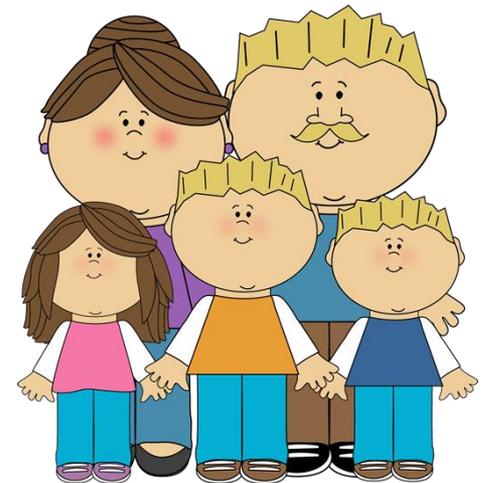
x	$g(x)$
-4	7
-1	1
2	-5
5	-11

For each comparison below, use the drop-down menu to select a symbol that correctly indicates the relationship between the first and the second quantity.

First Quantity	Comparison	Second Quantity
The y-coordinate of the y-intercept $f(x)$	<input type="text"/>	The y-coordinate of the y-intercept $g(x)$
$f(3)$	<input type="text"/>	$g(3)$
Maximum value of $f(x)$ on the interval $-5 \leq x \leq 5$	<input type="text"/>	Maximum value of $g(x)$ on the interval $-5 \leq x \leq 5$
$\frac{f(5) - f(2)}{5 - 2}$	<input type="text"/>	$\frac{g(5) - g(2)}{5 - 2}$

Function Families:

- Useful to group functions into families with similar patterns of change – families share certain general characteristics.



Function Families:

- Linear Functions (8.F.3, 8.F.4, F-IF.7a)*
- Exponential Functions (F-IF.8b, F-LE.1c, F-LE.2&3)*
- Quadratic functions (F-IF.7a, F-IF.8a, F-LE.1b)*
- Other (F-IF.7b)
- Inverse functions (F-BF.4a)

Functions in Context

- Qualitative interpretation (F-IF.2, 8.F.5, F-LE.5)
- Interpreting rate of change in context (8.F.4)
- Using appropriate vocabulary for features of graph (F-IF.4, F-BF.3)

Qualitative interpretation

- 8.F.5
- F-IF.2
- F-IF.4
- F-LE.5

Grade 7: Ratio & Proportion

2. Recognize and represent proportional relationships between quantities.
 - a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Grade 7: Ratio & Proportion

- c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*
- d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Unit rate \rightarrow Slope (8.EE.5)

- 8.EE.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Interpreting rate of change:

- 8.F.4
- F-LE.1b

Vocabulary

- Independent/dependent variables (6.EE.9)
- Increasing, Decreasing (8.F.5, F-IF.4)
- Linear, Non-Linear (8.F.5)
- Domain, Range (F-IF.1)
- Intercepts (F-IF.4)
- Relative maximums and minimums (F-IF.4)
- End Behavior (F-IF.4)
- Average Rate of Change (F-IF.6)
- Zeros (F-IF.8a)
- Even/Odd Functions (F-BF.3)



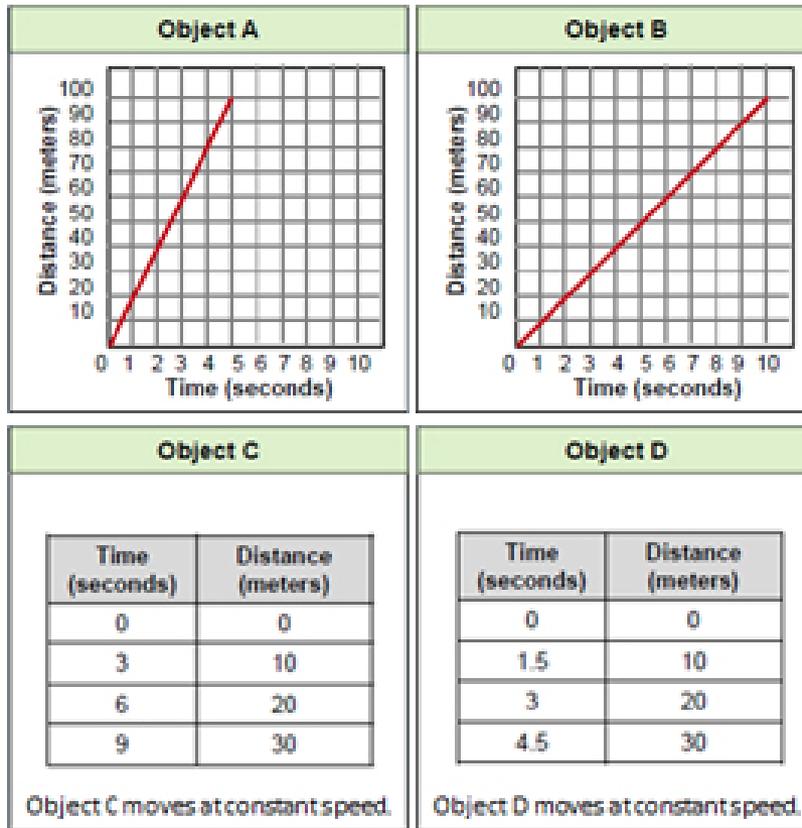
Linear Functions

- Recognizing Linearity
 - Constant rate of change
 - Equation $y=mx+b$ (8.F.3)
- Flexibility in Representations (8.F.2, F-IF.9)

- 
- How does the rate of change appear in different representations of a linear function?

PARCC items

SAMPLE ITEM



If an object has constant speed, then the speed can be computed by the change in distance divided by the change in time.

Information about objects A, B, C and D are shown. Objects C and D both have constant speed.

Based on the information given, drag and drop the object names in order from greatest speed to least speed in the table provided.

Object A	Greatest Speed Least Speed	
Object B		
Object C		
Object D		

Mathematical Practice 7

- By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.

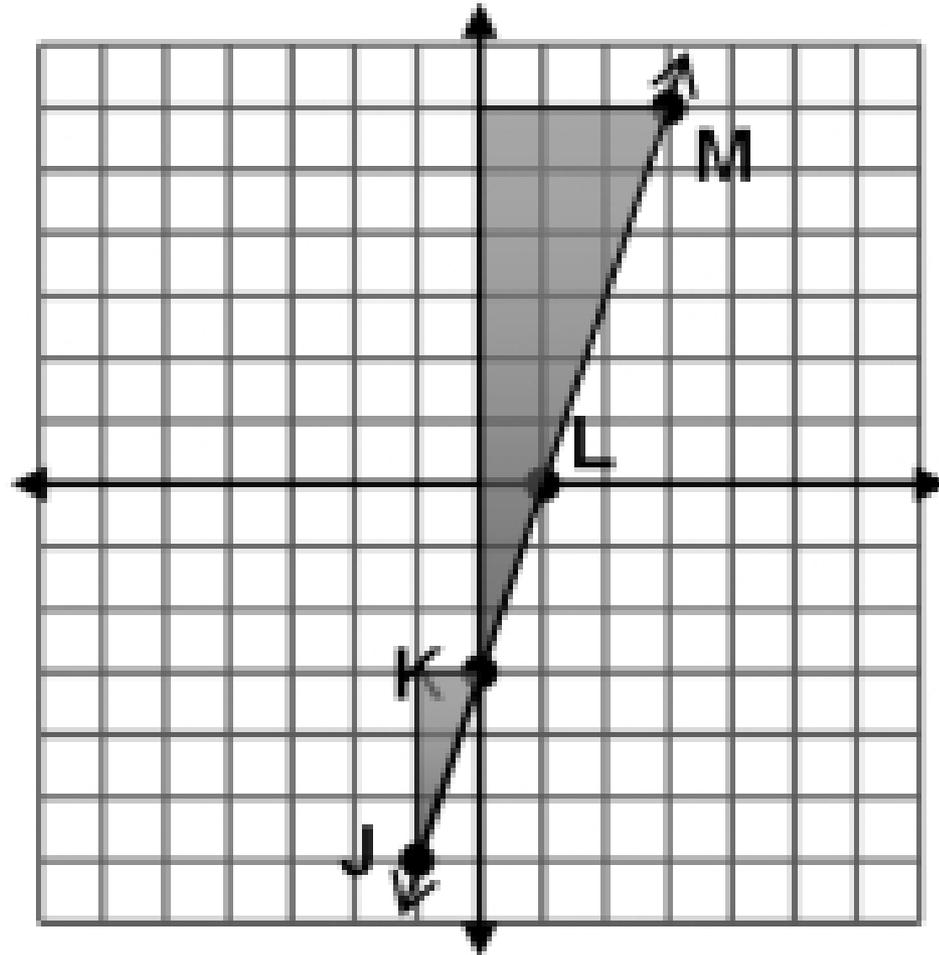
(CCSS (2010), p. 6)

“Slope triangles”

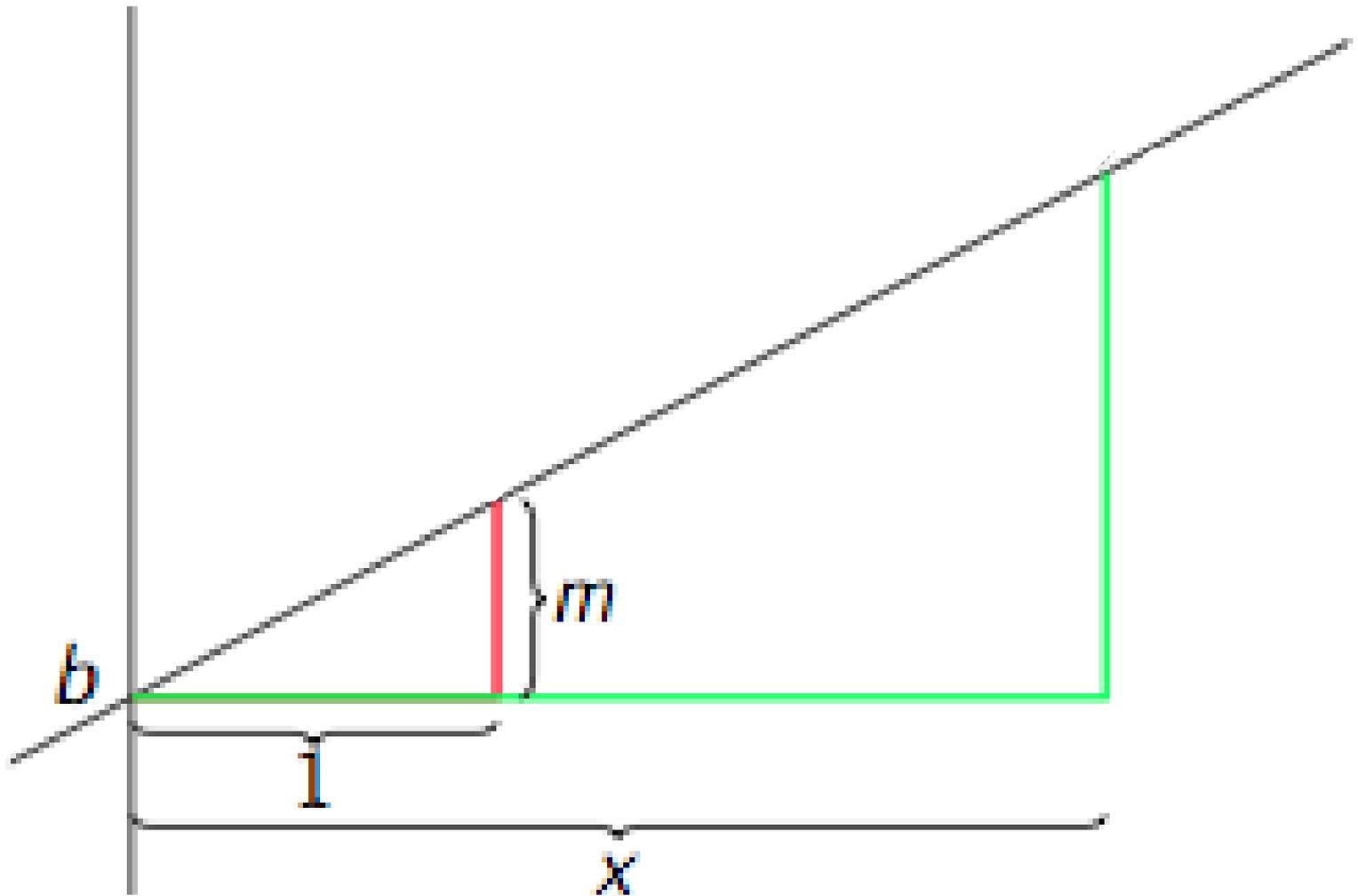
8.EE.6

- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ for a line intercepting the vertical axis at b .

“Slope triangles”



Slope triangles

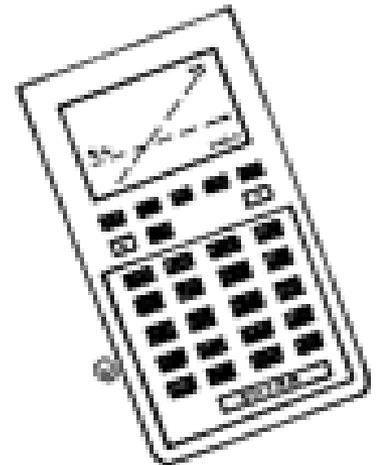


Modeling Linear Relationships

- 8.F.4
- F-LE.1a

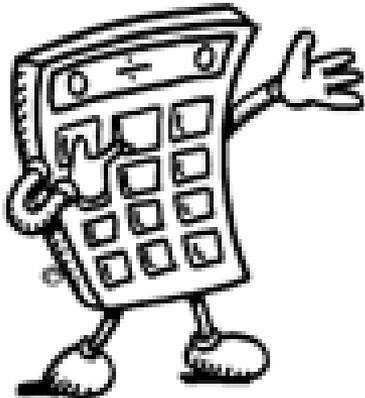
Transformations

- F-BF.3
- Observing changes in parameters m & b



Advantages of TECHNOLOGY

- increased emphasis on graphing
- able to quickly see multiple representations of function
- student involved in activity and discovery
- emphasizes global aspects of the graph

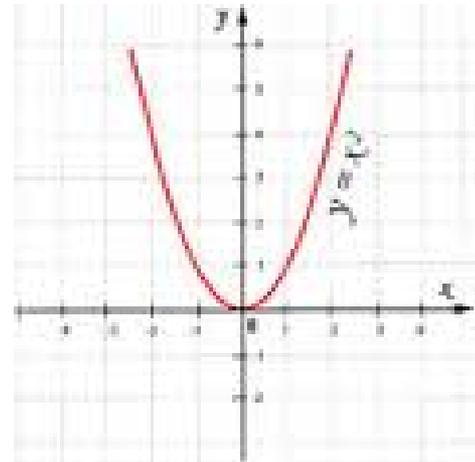


Exponential Functions

- F-IF.7e, F-IF.8b, F-LE.1c, F-LE.2&3
- Properties
 - Rate of change increases (growth) or decreases (decay) over the domain
- Representations

Quadratic Functions

- F-IF.7a, F-IF.8a, F-LE.1b
- rates of change that change at a constant rate
- Representations



Investigations with Non-Linear functions:

- F-BF.3
Transformations
- F-IF.6
Average rate of change
- F-BF.1a&b
Modeling

Other Functions

- F-IF.7b
- Properties
- Representations

Inverse Functions

- F-BF.4a
- Properties
- Representations

Function sense

- Develop flexibility in representations
- Use functions to model relationships

CONCLUSION:

When nature of problems we give changes,
so does nature of student reasoning!



Resources for tasks:

- <http://illustrativemathematics.org>
- <http://map.mathshell.org/materials/>
- <http://illuminations.nctm.org>
- <http://learnzillion.com/lessons>
- <http://parcconline.org>
- <http://www.smarterbalanced.org/sample-items-and-performance-tasks/>

Reflect

- In what ways has your understanding of functions changed?
- How will what you learned today affect your teaching?

