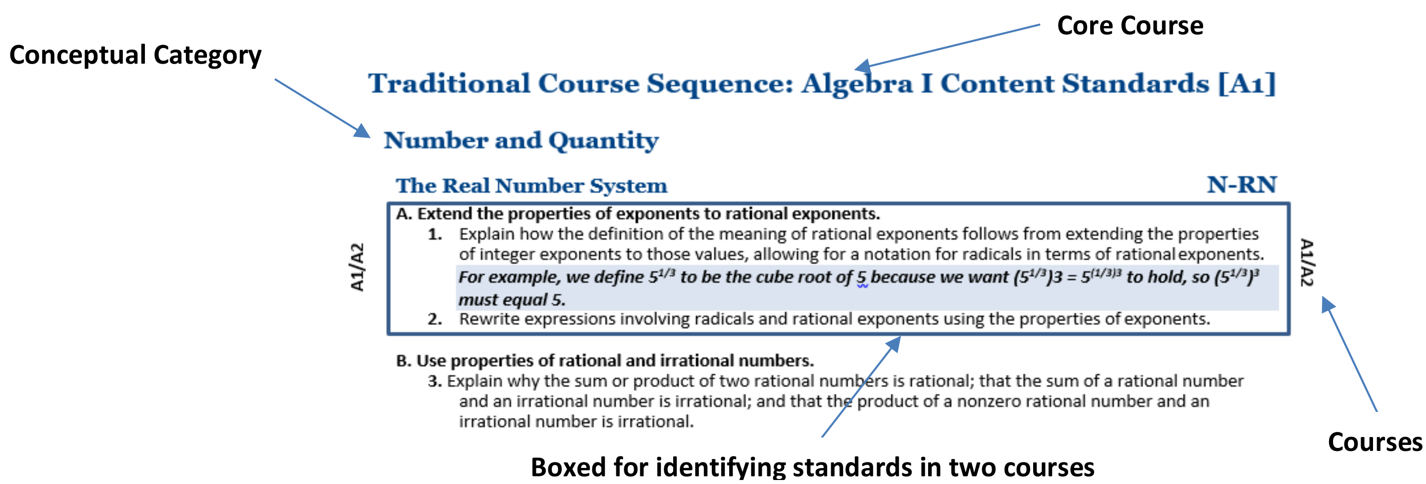
# RHODE ISLAND MODEL HIGH SCHOOL COURSE STANDARDS DISTRIBUTION FOR ALGEBRA 1, GEOEMTRY, AND ALGEBRA 2

Rhode Island Department of Education, 2021

# Mathematics Standards for High School (Organized by Model Course – Algebra 1, Geometry, Algebra 2)

The Rhode Island Core Standards for Mathematics (RICSM) are organized by grade level in grades K–8. At the high school level, the standards are organized by conceptual category: Number and Quantity, Algebra, Functions, Geometry, Modeling, and Probability and Statistics. These categories define the body of knowledge students should learn to be college- and career-ready and to be prepared to study more advanced topics in mathematics. Since the standards are organized by conceptual category, it becomes critically important to determine how they can be arranged into courses in order to provide a strong foundation for post-secondary success. To assist with this endeavor, RIDE has organized the RICSM for high school into the component model courses of the traditional Algebra 1, Geometry, and Algebra 2 (AGA) sequence. Over the course of the model AGA sequence all non-plus high school mathematics standards are included in at least one course. Some standards are included in two courses. These standards are boxed and labeled within each course. The standards should be instructed based on where they are included within the LEA's adopted high-quality curriculum. Plus standards (+) may be included in model AGA course content at the discretion of the LEA.




## Placement of the standards into courses and high-quality curriculum

In 2019, General Law (RIGL) § 16-22-32 was passed by the Rhode Island legislature. This legislation requires all Rhode Island LEAs to adopt high-quality curriculum materials for K- 12 mathematics that are aligned with the state's academic standards. Rhode Island published a list of approved high-quality curricula in mathematics in advance of the 2023 selection and adoption requirement for LEAs. The intent of this published list is to provide LEAs with the ability to choose a high-quality curriculum that best fits the needs of its students, teachers, and community. Different high-quality curricula vary in their placement of the high school standards into courses within the traditional AGA sequence. As indicated above, for standards that are included in multiple model courses, LEAs should implement those standards within the course(s) as defined within their adopted high-quality curriculum. For example, the enVision AGA program has identified N.RN.A.1 (noted in above graphic) in two courses, Algebra 1 and Algebra 2. A school that uses enVision would need to finalize what part of that standard is addressed in A1 and A2 based on how enVision covers that standard. Alternatively, a high-quality curriculum may address one of the boxed standards in a single course. If this is the case, there is no need to adjust instruction so the standard is taught in both A1 and A2.

**NOTE:** Over the course of the AGA sequence all non-plus high school mathematics standards are to be instructed. Plus standards (+) may be included in AGA course content at the discretion of the LEA.

## Organization of High School Courses

Each model high school course is presented in three sections:

- An introduction and description of the critical areas for learning in that course.
- An overview listing the conceptual categories, domains, and clusters included in that course.
- The content standards, organized by conceptual category, domain, and cluster heading. Note: Those standards that may be multi-course are boxed and labeled.

# High School Model Mathematics Course: Algebra 1[A1]

## Introduction

The fundamental purpose of the Model Algebra I course is to formalize and extend the mathematics that students learned in the middle grades. For the high school Model Algebra I course, instructional time should focus on four critical areas: (1) deepen and extend understanding of linear and exponential relationships; (2) contrast linear and exponential relationships with each other and engage in methods for analyzing, solving, and using quadratic functions; (3) extend the laws of exponents to square and cube roots; and (4) apply linear models to data that exhibit a linear trend.

1. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.
2. In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and exponential functions, including sequences, and also explore absolute value functions; they interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students build on and extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
3. Students extend the laws of exponents to rational exponents involving square and cube roots and apply this new understanding of number; they strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students become facile with algebraic manipulation, including rearranging and collecting terms and factoring. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions like absolute value.
4. Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

# Traditional Course: Model Algebra I Overview [A1]

## Number and Quantity

**The Real Number System**

A. Extend the properties of exponents to rational exponents.
B. Use properties of rational and irrational numbers.

**Quantities**

A. Reason quantitatively and use units to solve problems.

## Algebra

**Seeing Structure in Expressions**

A. Interpret the structure of linear, quadratic, and exponential expressions with integer exponents.
B. Write expressions in equivalent forms to solve problems.

**Arithmetic with Polynomials and Rational Expressions**

A. Perform arithmetic operations on polynomials.

**Creating Equations**

A. Create equations that describe numbers or relationships.

**Reasoning with Equations and Inequalities**

A. Understand solving equations as a process of reasoning and explain the reasoning.
B. Solve equations and inequalities in one variable.
C. Solve systems of equations.
D. Represent and solve equations and inequalities graphically.

### *Standards for Mathematical Practice*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Function

**Interpreting Functions**

A. Understand the concept of a function and use function notation.
B. Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context.
C. Analyze functions using different representations.

**Building Functions**

A. Build a function that models a relationship between two quantities.
B. Build new functions from existing functions.

**Linear, Quadratic, and Exponential Models**

A. Construct and compare linear, quadratic, and exponential models and solve problems.
B. Interpret expressions for functions in terms of the situation they model.

## Statistics and Probability

**Interpreting Categorical and Quantitative Data**

A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.
B. Summarize, represent, and interpret data on two categorical and quantitative variables.
C. Interpret linear models.

# Traditional Course: Model Algebra I Content Standards [A1]

## Number and Quantity

### The Real Number System — N-RN

A1/A2

**A. Extend the properties of exponents to rational exponents.**

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

   ***For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.***

A1/A2

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**B. Use properties of rational and irrational numbers.**

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### Quantities — N-Q

**A. Reason quantitatively and use units to solve problems.**

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★
2. Define appropriate quantities for the purpose of descriptive modeling.★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★
   - **a.** Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure.★

## Algebra

### Seeing Structure in Expressions — A-SSE

A1/A2

**A. Interpret the structure of linear, quadratic, exponential, polynomial, and rational expressions.**

1. Interpret expressions that represent a quantity in terms of its context.★
   - **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
   - ***b.*** Interpret complicated expressions by viewing one or more of their parts as a single entity.

   ***For example, interpret $P(1 + r)^n$ as the product of $P$ and a factor not depending on $P$.***
2. Use the structure of an expression to identify ways to rewrite it.

   ***For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.***

A1/A2

**B. Write expressions in equivalent forms to solve problems.**

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   - a. Factor a quadratic expression to reveal the zeros of the function it defines.
   - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   - c. Use the properties of exponents to transform expressions for exponential functions.

   ***For example, the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.***

## Arithmetic with Polynomials and Rational Expressions A-APR

A1/A2

**A. Perform arithmetic operations on polynomials.**

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.
   - **a.** Perform operations on polynomial expressions (addition, subtraction, multiplication, division) and compare the system of polynomials to the system of integers when performing operations.
   - **b.** Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.

A1/A2

## Creating Equations A-CED

A1/A2

**A. Create equations that describe numbers or relationships.**

1. Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear and quadratic functions, and simple root and rational functions and exponential functions.) ★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. ★

   ***For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.***
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

   ***For example, rearrange Ohm's law $V = IR$ to highlight resistance, $R$.***

A1/A2

## Reasoning with Equations and Inequalities A-REI

A1/A2

**A. Understand solving equations as a process of reasoning and explain the reasoning.**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

A1/A2

**B. Solve equations and inequalities in one variable.**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
   - a. Solve linear equations and inequalities in one variable involving absolute value.
4. Solve quadratic equations in one variable.
   - a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**C. Solve systems of equations.**

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A1/A2

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

   ***For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.***

A1/A2

**D. Represent and solve equations and inequalities graphically.**

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an

equation in two variables is a solution to the equation.

A1/A2

**11.** Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

A1/A2

**12.** Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

# Functions

## Interpreting Functions — F-IF

**A. Understand the concept of a function and use function notation.**

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

   ***For example, given a function representing a car loan, determine the balance of the loan at different points in time.***

A1/A2

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

   ***For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.***

**B. Interpret functions that arise in applications in terms of the context (linear, quadratic, exponential, rational, polynomial, square root, cube root, trigonometric, logarithmic).**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

   ***For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

A1/A2

**C. Analyze functions using different representations.**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.★

A1/A2

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.★

A1/A2

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph,

and interpret these in terms of a context.

**b.** Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase. $V_n = P(1+r)^n$

***For example, identify percent rate of change in functions such as y = $(1.02)^t$, y = $(0.97)^t$, y = $(1.01)^{12t}$, and y = $(1.2)^{t/10}$, and classify them as representing exponential growth or decay.***

A1/A2

A1/A2

**9.** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

***For example, given a graph of one polynomial function (including quadratic functions) and an algebraic expression for another, say which has the larger/smaller relative maximum and/or minimum.***

**10.** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

## Building Functions — F-BF

**A. Build a function that models a relationship between two quantities.**

**1.** Write a function (linear, quadratic, exponential, simple rational, radical, logarithmic, and trigonometric) that describes a relationship between two quantities.★

**a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.★

**b.** Combine standard function types using arithmetic operations.★

***For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.***

**2.** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

**B. Build new functions from existing functions.**

**3.** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.)

**4.** Find inverse functions algebraically and graphically.

**a.** Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. (Include linear and simple polynomial, rational, and exponential functions.)

***For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.***

## Linear, Quadratic, and Exponential Models — F-LE

**A. Construct and compare linear, quadratic, and exponential models and solve problems.**

**1.** Distinguish between situations that can be modeled with linear functions and with exponential functions.★

**a.** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.★

**b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.★

**c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.★

**2.** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).★

**3.** Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

**B. Interpret expressions for functions in terms of the situation they model.**

**5.** Interpret the parameters in a linear or exponential function (of the form $f(x) = b^x + k$) in terms of a context.★

# Statistics and Probability

## Interpreting Categorical and Quantitative Data S-ID

**A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.**

**1.** Represent data with plots on the real number line (dot plots, histograms, and box plots).★

**2.** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★

**3.** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

**B. Summarize, represent, and interpret data on two categorical and quantitative variables.**

**5.** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

**6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

- **a.** Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use functions fitted to data or choose a function suggested by the context. Emphasize linear and exponential models.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.★
- **c.** Fit a linear function for a scatter plot that suggests a linear association.★

**C. Interpret linear models.**

**7.** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

**8.** Compute (using technology) and interpret the correlation coefficient of a linear fit.★

**9.** Distinguish between correlation and causation.★

# High School Model Mathematics Course: Geometry [GEO]

## Introduction

The fundamental purpose of the Model Geometry course is to formalize and extend students' geometric experiences from the middle grades. In this high school Model Geometry course, students explore more complex geometric situations and deepen their explanations of geometric relationships by presenting and hearing formal mathematical arguments. Important differences exist between this course and the historical approach taken in geometry classes. For example, transformations are emphasized in this course. Close attention should be paid to the introductory content for the Geometry conceptual category.

For the high school Model Geometry course, instructional time should focus on six critical areas: (1) establish criteria for congruence of triangles based on rigid motions; (2) establish criteria for similarity of triangles based on dilations and proportional reasoning; (3) informally develop explanations of circumference, area, and volume formulas; (4) apply the Pythagorean Theorem to the coordinate plane; (5) prove basic geometric theorems; and (6) extend work with probability.

1. Students have prior experience with drawing triangles based on given measurements and performing rigid motions including translations, reflections, and rotations. They have used these to develop notions about what it means for two objects to be congruent. In this course, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems— using a variety of formats including deductive and inductive reasoning and proof by contradiction—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.
2. Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on their work with quadratic equations done in Model Algebra I. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.
3. Students' experience with three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.
4. Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use the rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals, and slopes of parallel and perpendicular lines, which relates back to work done in the Model Algebra I course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.
5. Students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations—which relates back to work done in the Model Algebra I course—to determine intersections between lines and circles or parabolas and between two circles.
6. Building on probability concepts that began in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

# Traditional Course: Model Geometry Overview [G]

## Geometry

**Congruence**

- **A.** Experiment with transformations in the plane.
- **B.** Understand congruence in terms of rigid motions.
- **C.** Prove geometric theorems and, when appropriate, the converse of theorems.
- **D.** Make geometric constructions.

**Similarity, Right Triangles, and Trigonometry**

- **A.** Understand similarity in terms of transformations.
- **B.** Prove theorems involving similarity.
- **C.** Define trigonometric ratios and solve problems involving right triangles.

**Circles**

- **A.** Understand and apply theorems about circles.
- **B.** Find arc lengths and area of sectors of circles.

**Expressing Geometric Properties with Equations**

- **A.** Translate between the geometric description and the equation for a conic section.
- **B.** Use coordinates to prove simple geometric theorems algebraically.

**Geometric Measurement and Dimension**

- **A.** Explain volume formulas and use them to solve problems.
- **B.** Visualize relationships between two-dimensional and three-dimensional objects.

**Modeling with Geometry**

- **A.** Apply geometric concepts in modeling situations.

***Standards for Mathematical Practice***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Statistics and Probability

**Conditional Probability and the Rules of Probability**

- **A.** Understand independence and conditional probability and use them to interpret data from simulations or experiments.
- **B.** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

# Traditional Course: Model Geometry Content Standards [G]

## Geometry

### Congruence G-CO

**A. Experiment with transformations in the plane.**

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**B. Understand congruence in terms of rigid motions.**

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

**C. Prove geometric theorems and, when appropriate, the converse of theorems.**

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.
11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
    a. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.

**D. Make geometric constructions.**

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

## Similarity, Right Triangles, and Trigonometry G-SRT

**A. Understand similarity in terms of similarity transformations.**

1. Verify experimentally the properties of dilations given by a center and a scale factor:
    a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.

**B. Prove theorems involving similarity.**

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**C. Define trigonometric ratios and solve problems involving right triangles.**

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

## Circles G-C

**A. Understand and apply theorems about circles.**

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.

**B. Find arc lengths and areas of sectors of circles.**

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## Expressing Geometric Properties with Equations G-GPE

**A. Translate between the geometric description and the equation for a conic section.**

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G/A2

2. Derive the equation of a parabola given a focus and directrix.

G/A2

**B. Use coordinates to prove simple geometric theorems algebraically.**

4. Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem.
   ***For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).***
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).★

## Geometric Measurement and Dimension — G-GMD

**A. Explain volume formulas and use them to solve problems.**

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

**B. Visualize relationships between two-dimensional and three-dimensional objects.**

4. Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

## Modeling with Geometry — G-MG

**A. Apply geometric concepts in modeling situations.**

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★
4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense.★

# Statistics and Probability

## Conditional Probability and the Rules of Probability — S-CP

G/A2

**A. Understand independence and conditional probability and use them to interpret data from simulations or experiments.**

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★

G/A2

G/A2

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ★

***For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.***

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. ★

***For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.***

**B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

6. Find the conditional probability of $A$ given $B$ as the fraction of $B$'s outcomes that also belong to $A$, and interpret the answer in terms of the model. ★
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model. ★

G/A2

# High School Model Mathematics Course: Algebra 2 [A2]

## Introduction

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include logarithmic, polynomial, rational, piecewise and radical functions in the Model Algebra II course. Students work closely with the expressions that define the functions, are facile with algebraic manipulations of expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

For the high school Model Algebra II course, instructional time should focus on four critical areas: (1) relate arithmetic of rational expressions to arithmetic of rational numbers; (2) expand understandings of functions and graphing to include trigonometric functions; (3) synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions; and (4) relate data display and summary statistics to probability and explore a variety of data collection methods.

1. A central theme of this Model Algebra II course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined.
2. Building on their previous work with functions and on their work with trigonometric ratios and circles in the Model Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena.
3. Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this Model Algebra II course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.
4. Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

# Traditional Course: Model Algebra 2 Overview [A2]

## Number and Quantity

**The Real Number System**

- **A.** Extend the properties of exponents to rational exponents.
- **B.** Use properties of rational and irrational numbers.

**Quantities**

- **A.** Reason quantitatively and use units to solve problems.

## Algebra

**Seeing Structure in Expressions**

- **A.** Interpret the structure of linear, quadratic, and exponential expressions with integer exponents.
- **B.** Write expressions in equivalent forms to solve problems.

**Arithmetic with Polynomials and Rational Expressions**

- **A.** Perform arithmetic operations on polynomials.

**Creating Equations**

- **A.** Create equations that describe numbers or relationships.

**Reasoning with Equations and Inequalities**

- **A.** Understand solving equations as a process of reasoning and explain the reasoning.
- **B.** Solve equations and inequalities in one variable.
- **C.** Solve systems of equations.
- **D.** Represent and solve equations and inequalities graphically.

***Standards for Mathematical Practice***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Function

**Interpreting Functions**

- **A.** Understand the concept of a function and use function notation.
- **B.** Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context.
- **C.** Analyze functions using different representations.

**Building Functions**

- **A.** Build a function that models a relationship between two quantities.
- **B.** Build new functions from existing functions.

**Linear, Quadratic, and Exponential Models**

- **A.** Construct and compare linear, quadratic, and exponential models and solve problems.
- **B.** Interpret expressions for functions in terms of the situation they model.

## Statistics and Probability

**Interpreting Categorical and Quantitative Data**

- **A.** Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.
- **B.** Summarize, represent, and interpret data on two categorical and quantitative variables.
- **C.** Interpret linear models.

# Traditional Course: Model Algebra 2 Content Standards [A2]

## Number and Quantity [N]

A1/A2

### The Real Number System — N-RN

**A. Extend the properties of exponents to rational exponents.**

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
   ***For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.***
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### The Complex Number System — N-CN

**A. Perform arithmetic operations with complex numbers.**

1. Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.
2. Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.

**B. Use complex numbers in polynomial identities and equations.**

7. Solve quadratic equations with real coefficients that have complex solutions.

A1/A2

## Algebra [A]

A1/A2

### Seeing Structure in Expressions — A-SSE

**A. Interpret the structure of linear, quadratic, exponential, polynomial, and rational expressions.**

1. Interpret expressions that represent a quantity in terms of its context.★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   *b.* Interpret complicated expressions by viewing one or more of their parts as a single entity.
   ***For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.***
2. Use the structure of an expression to identify ways to rewrite it.
   ***For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.***

A1/A2

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.★
   ***For example, calculate mortgage payments.***

A1/A2

### Arithmetic with Polynomials and Rational Expressions — A-APR

**A. Perform arithmetic operations on polynomials.**

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.
   a. Perform operations on polynomial expressions (addition, subtraction, multiplication, division) and compare the system of polynomials to the system of integers when performing operations.
   b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.

A1/A2

**B. Understand the relationship between zeros and factors of polynomials.**

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**C. Use polynomial identities to solve problems.**

4. Prove polynomial identities and use them to describe numerical relationships.
   ***For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.***

**D. Rewrite rational expressions.**

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

## Creating Equations — A-CED

A1/A2

**A. Create equations that describe numbers or relationships.**

1. Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear and quadratic functions, and simple root and rational functions and exponential functions.)★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.★
   ***For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.***
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★
   ***For example, rearrange Ohm's law $V = IR$ to highlight resistance, $R$.***

## Reasoning with Equations and Inequalities — A-REI

**A. Understand solving equations as a process of reasoning and explain the reasoning.**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

A1/A2

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A1/A2

**C. Solve systems of equations.**

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
   ***For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.***

**D. Represent and solve equations and inequalities graphically.**

11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

A1/A2

# Functions [F]

A1/A2

## Interpreting Functions F-IF

**A. Understand the concept of a function and use function notation.**

**3.** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

***For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.***

**B. Interpret functions that arise in applications in terms of the context (linear, quadratic, exponential, rational, polynomial, square root, cube root, trigonometric, logarithmic).**

**4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

**5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

***For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***

**6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**C. Analyze functions using different representations.**

**7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★

A1/A2

- **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★

A1/A2

- **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.★

**9.** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

***For example, given a graph of one polynomial function (including quadratic functions) and an algebraic expression for another, say which has the larger/smaller relative maximum and/or minimum.***

A1/A2

**10.** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

A1/A2

## Building Functions F-BF

**A. Build a function that models a relationship between two quantities.**

**1.** Write a function (linear, quadratic, exponential, simple rational, radical, logarithmic, and trigonometric) that describes a relationship between two quantities.★

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.★
- **b.** Combine standard function types using arithmetic operations.★

***For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.***

**2.** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

A1/A2

A1/A2

**B. Build new functions from existing functions.**

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.)
4. Find inverse functions algebraically and graphically.
   a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. (Include linear and simple polynomial, rational, and exponential functions.)

   ***For example, $f(x) = 2x^3$ or $f(x) = {}^{(x+1)}/_{(x-1)}$ for $x \neq 1$.***

A1/A2

## Linear, Quadratic, and Exponential Models — F-LE

**A. Construct and compare linear, quadratic, and exponential models and solve problems.**

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

## Trigonometric Functions — F-TF

**A. Extend the domain of trigonometric functions using the unit circle.**

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**B. Model periodic phenomena with trigonometric functions.**

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

# Statistics and Probability

G/A2

## Conditional Probability and the Rules of Probability — S-CP

**A. Understand independence and conditional probability and use them to interpret data from simulations or experiments.**

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.★

   ***For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.***
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★

***For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.***

G/A2

G/A2

**B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

**8.** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.★

**9.** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.★

G/A2